APPLICATION OF THE EIGENMODE TRANSFORMATION TECHNIQUE FOR THE ANALYSIS OF PLANAR TRANSMISSION LINES

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ABSTRACT

The eigenmode transformation technique is suitable for the analysis of inhomogeneously filled shielded waveguides containing metal inserts. The permittivity of the filling medium may be an arbitrary function of the transverse coordinates. The method is based on expanding the electromagnetic field in terms of the eigenmodes of the corresponding empty shielding waveguide. The metal inserts have the effect of linearly transforming these eigenmodes into those of the waveguide containing the metal inserts only. This leads to a proper matrix eigenvalue problem. The method is applied to different types of planar transmission lines and the results are compared with other methods.

INTRODUCTION

We consider the inhomogeneously filled waveguide shown in Fig. 1. The inhomogeneity is described by a permittivity which may be an arbitrary function of the transverse coordinates. Just one metal insert will be considered here. The extension to more than one metal insert is straightforward.

Although the method presented in [1] leads to a proper matrix eigenvalue problem, to the authors' best knowledge it has not yet been applied to the actual computation of eigenmodes. As has been shown in [2], the set of TE- and TM-eigenmodes corresponding to the empty shielding waveguide is complete if no a-priori-coupling between the transversal and axial field components of these eigenmodes is assumed. On the other hand, for the waveguides with metal inserts, additionally to the TE- and TM-eigenmodes one ore more TEM-eigenmodes, corresponding to the number of metal inserts, have to be taken into account to form a complete set. In this contribution we will follow the analysis of [1] by expanding the field of the inhomogeneously filled waveguide in terms of the eigenmodes of the corresponding empty waveguide. The influence of metal inserts can however be described by a linear transformation of the matrices which constitute the eigenvalue problem corresponding to the inhomogeneously filled waveguide without metal inserts. This results in a new proper matrix eigenvalue problem. The computation of the transformation matrices can be treated as a separate problem ([3], [4], [5]) what leads to a modular character of the method.

THEORY

Refering to Fig. 1, the cross section (contour) of the shielding waveguide and that of the metal insert are denoted by S (C) and S_0 (C_0), respectively. The unit vector normal to both, C and C_0, is denoted by $\hat{\boldsymbol{n}}$. The permittivity ϵ_r is a function of the transverse coordinates $\boldsymbol{r}$. The direction of propagation, in which the structure is uniform, is taken along the z-axis with a corresponding propagation constant β.

Let $\{\mathcal{H}_{zn}\}$ and $\{\mathcal{E}_{zn}\}$ be the complete sets of axial magnetic and axial electric fields characterizing the TE- and TM-eigenmodes, respectively, corresponding to the waveguide of Fig. 1 without dielectric. It can be easily shown that the TEM-eigennmode (due to the metal insert) is in fact a TM-eigenmode with vanishing cutoff wavenumber. Therefore the TEM-eigenmode will be formally included into the set of TM-eigenmodes for the sake of simplicity. The axial fields $\mathcal{H}_{zn}$ and $\mathcal{E}_{zn}$ are real functions of the transverse coordinates and correspond to cut-off wavenumbers κ_n^h and κ_n^e, respectively. They satisfy the orthogonality relations

$$\int_{S-S_0} \boldsymbol{\nabla}_t \mathcal{H}_{zn} \boldsymbol{\nabla}_t \mathcal{H}_{zm}\, dS = \delta_{nm} \quad , \qquad (1\text{-a})$$

$$\int_{S-S_0} \boldsymbol{\nabla}_t \mathcal{E}_{zn} \boldsymbol{\nabla}_t \mathcal{E}_{zm}\, dS = \delta_{nm} \quad , \qquad (1\text{-b})$$

where δ_{nm} is the Kronecker delta and $\boldsymbol{\nabla}_t$ is the transverse component of the del-operator. Note that $\mathcal{H}_{zn}$ and $\mathcal{E}_{zn}$ are defined over $S - S_0$ only.

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Let $\hat{\boldsymbol{k}}$ be the unit vector in axial direction. The sets $\{\nabla_t \mathcal{E}_{zn}\}$ and $\left\{\nabla_t \mathcal{H}_{zn} \times \hat{\boldsymbol{k}}\right\}$ are complete with respect to curl-free and divergence-free transverse electric fields, respectively. The sets $\{\nabla_t \mathcal{H}_{zn}\}$ and $\left\{\hat{\boldsymbol{k}} \times \nabla_t \mathcal{E}_{zn}\right\}$ have the same properties with respect to the transverse magnetic field. Let $\boldsymbol{E}_t$, $\boldsymbol{H}_t$, E_z, and H_z be the transverse electric, transverse magnetic, axial electric, and axial magnetic field components, respectively, then we can write

$$\epsilon_r \boldsymbol{E}_t = \sum_n^{\infty} U_n^H \left(\nabla_t \mathcal{H}_{zn} \times \hat{\boldsymbol{k}}\right) + \sum_n^{\infty} U_n^E \nabla_t \mathcal{E}_{zn} \quad \text{(2-a)}$$

$$\boldsymbol{H}_t = \sum_n^{\infty} I_n^H \nabla_t \mathcal{H}_{zn} + \sum_n^{\infty} I_n^E \left(\hat{\boldsymbol{k}} \times \nabla_t \mathcal{E}_{zn}\right) \quad \text{(2-b)}$$

$$\epsilon_r E_z = \sum_n^{\infty} U_n^L \kappa_n^e \mathcal{E}_{zn} \quad , \quad \text{(2-c)}$$

$$H_z = \sum_n^{\infty} I_n^L \kappa_n^h \mathcal{H}_{zn} \quad . \quad \text{(2-d)}$$

where the z-dependence $e^{-j\beta z}$ has been dropped out for all field components.

Let $\{h_{zn}\}$ and $\{e_{zn}\}$ denote the complete sets of axial magnetic and axial electric fields characterizing the TE- and TM-eigenmodes, respectively, which correspond to the empty waveguide. For h_{zn} and e_{zn} with corresponding cut-off wavenumbers k_n^h and k_n^e, respectively, orthogonality relations corresponding to (1-a) and (1-b) are valid. Inside a metal insert, the electromagnetic field vanishes. Since e_{zn} and h_{zn} are defined everywhere over S we can expand the fields which are given by $\nabla_t \mathcal{E}_{zi}$, $\nabla_t \mathcal{H}_{zi}$, $\mathcal{E}_{zi}$, and $\mathcal{H}_{zi}$ over $S - S_0$ and by zero over S_0 with respect to $\nabla_t e_{zp}$, $\left(\nabla_t h_{zp} \text{ and } \left(\hat{\boldsymbol{k}} \times \nabla_t e_{zp}\right)\right)$, e_{zp}, and h_{zp}, respectively [3].

$$\sum_p^{\infty} \mathcal{U}_{pi}^{EE} \nabla_t e_{zp} = \begin{cases} \nabla_t \mathcal{E}_{zi} & \text{over } S - S_0 \\ 0 & \text{over } S_0 \end{cases} \quad \text{(3-a)}$$

$$\sum_p^{\infty} \mathcal{J}_{pi}^{HH} \nabla_t h_{zp} + \sum_p^{\infty} \mathcal{J}_{pi}^{EH} \left(\hat{\boldsymbol{k}} \times \nabla_t e_{zp}\right) = \begin{cases} \nabla_t \mathcal{H}_{zi} & \text{over } S - S_0 \\ 0 & \text{over } S_0 \end{cases} \quad \text{(3-b)}$$

$$\sum_p^{\infty} \mathcal{U}_{pi}^{L} k_p^e e_{zp} = \begin{cases} \kappa_i^e \mathcal{E}_{zi} & \text{over } S - S_0 \\ 0 & \text{over } S_0 \end{cases} \quad \text{(3-c)}$$

$$\sum_p^{\infty} \mathcal{J}_{pi}^{L} k_p^h h_{zp} = \begin{cases} \kappa_i^h \mathcal{H}_{zi} & \text{over } S - S_0 \\ 0 & \text{over } S_0 \end{cases} \quad \text{(3-d)}$$

Let us denote the field given by the right-hand side of (3-b) by $\mathcal{H}_t$. Because the tangential component of $\mathcal{H}_t$ has a step discontinuity at C_0, $(\nabla_t \times \mathcal{H}_t)$ which includes the normal derivative of the tangential component behaves as a Dirac delta function there. This Dirac delta function is just the axial component of the surface current at C_0. The vector $(\nabla_t \times \mathcal{H}_t)$ can then vanish everywhere over S except at C_0, and hence $\mathcal{H}_t$ cannot be expanded in terms of the curl-free set $\{\nabla_t h_{zp}\}$ only. It needs, in addition, the divergence-free set $\left\{\hat{\boldsymbol{k}} \times \nabla_t e_{zp}\right\}$. On the other hand, the functions defined by the right-hand side of (3-a) can be expanded in terms of the curl-free set $\{\nabla_t e_{zp}\}$ because the tangential component of this function is continuous across C_0.

Substituting the field representations according to (2-a)–(2-d) into Maxwell's equations and making use of the orthogonality properties according to (1-a)– (1-b), one arrives at

$$\sum_i^{\infty} \mathcal{T}_{in} U_i^E + \sum_i^{\infty} \mathcal{R}_{ni}^h U_i^H = \frac{-jZ_0}{\tilde{\kappa}_n^h} I_n^L, \quad \text{(4-a)}$$

$$I_n^E = \frac{-j}{\tilde{\kappa}_n^e Z_0} U_n^L, \quad \text{(4-b)}$$

$$\frac{\tilde{\kappa}_n^e}{j\tilde{\beta}} \sum_i^{\infty} \mathcal{S}_{ni} U_i^L + \sum_i^{\infty} \mathcal{R}_{ni}^e U_i^E + \sum_i^{\infty} \mathcal{T}_{ni} U_i^H = \frac{Z_0}{\tilde{\beta}} I_n^E. \quad \text{(4-c)}$$

$$\sum_i^{\infty} \mathcal{T}_{in} U_i^E + \sum_i^{\infty} \mathcal{R}_{ni}^h U_i^H = \frac{Z_0}{\tilde{\beta}} I_n^H \quad , \quad \text{(4-d)}$$

$$\tilde{\kappa}_n^h I_n^L + j\tilde{\beta} I_n^H = \frac{j}{Z_0} U_n^H \quad \text{(4-e)}$$

$$\tilde{\beta} I_n^E = \frac{1}{Z_0} U_n^E \quad , \quad \text{(4-f)}$$

where Z_0 denotes the intrinsic impedance of free space. Quantities marked with a tilde ($\tilde{}$) are normalized to the free space wavenumber k_0. The coupling integrals $\mathcal{R}_{ij}^h$, $\mathcal{R}_{ij}^e$, $\mathcal{T}_{ij}$ and $\mathcal{S}_{ij}$ read

$$\mathcal{R}_{ij}^h = \int_{S-S_0} \epsilon_r^{-1} \nabla_t \mathcal{H}_{zi} \cdot \nabla_t \mathcal{H}_{zj} \, dS \quad , \quad \text{(5-a)}$$

$$\mathcal{R}_{ij}^e = \int_{S-S_0} \epsilon_r^{-1} \nabla_t \mathcal{E}_{zi} \cdot \nabla_t \mathcal{E}_{zj} \, dS \quad , \quad \text{(5-b)}$$

$$\mathcal{T}_{ij} = \int_{S-S_0} \epsilon_r^{-1} \left(\nabla_t \mathcal{E}_{zi} \times \nabla_t \mathcal{H}_{zj}\right) \cdot \hat{\boldsymbol{k}} \, dS \quad , \quad \text{(5-c)}$$

$$\mathcal{S}_{ij} = \kappa_i^e \kappa_j^e \int_{S-S_0} \epsilon_r^{-1} \mathcal{E}_{zi} \mathcal{E}_{zj} \, dS \quad . \quad \text{(5-d)}$$

Substituting the series representations for $\nabla_t \mathcal{H}_{zi}$, $\nabla_t \mathcal{E}_{zi}$ and $\mathcal{E}_{zi}$ according to (3-a)–(3-c) into (5-a)–(5-d), the integrals can be extended over S. This is possible because the series are defined over the whole S and vanish over S_0. In matrix notation, we get the coupling matrices $[\mathcal{R}^h]$, $[\mathcal{R}^e]$, $[\mathcal{T}]$ and $[\mathcal{S}]$ with the elements $\mathcal{R}_{ij}^h$,

$\mathcal{R}^e_{ij}$, $\mathcal{T}_{ij}$ and $\mathcal{S}_{ij}$, respectively, as a linear transformation of the matrices $[R^h]$, $[R^e]$, $[T]$, and $[S]$.

$$[\mathcal{R}^h] = [[\mathcal{J}^{HH}]^t, [\mathcal{J}^{EH}]^t] \begin{bmatrix} [R^h] & [T]^t \\ [T] & [R^e] \end{bmatrix} \begin{bmatrix} [\mathcal{J}^{HH}] \\ [\mathcal{J}^{EH}] \end{bmatrix} \tag{6-a}$$

$$[\mathcal{R}^e] = [\mathcal{U}^{EE}]^t \, [R^e] \, [\mathcal{U}^{EE}] \tag{6-b}$$

$$[\mathcal{T}] = [\mathcal{U}^{EE}]^t \, [[T], [R^e]] \begin{bmatrix} [\mathcal{J}^{HH}] \\ [\mathcal{J}^{EH}] \end{bmatrix} \tag{6-c}$$

$$[\mathcal{S}] = [\mathcal{U}^L]^t \, [S] \, [\mathcal{U}^L] \tag{6-d}$$

The elements of $[R^h]$, $[R^e]$, $[T]$, and $[S]$ are given by

$$R^h_{ij} = \int_S \epsilon_r^{-1} \nabla_t h_{zi} \cdot \nabla_t h_{zj} \, dS \quad , \tag{7-a}$$

$$R^e_{ij} = \int_S \epsilon_r^{-1} \nabla_t e_{zi} \cdot \nabla_t e_{zj} \, dS \quad , \tag{7-b}$$

$$T_{ij} = \int_S \epsilon_r^{-1} \left(\nabla_t e_{zi} \times \nabla_t h_{zj} \right) \cdot \hat{\boldsymbol{k}} \, dS \quad , \tag{7-c}$$

$$S_{ij} = k^e_i k^e_j \int_S \epsilon_r^{-1} e_{zi} e_{zj} \, dS \quad . \tag{7-d}$$

The matrices $[R^h]$, $[R^e]$, $[T]$, and $[S]$ characterize the dielectric coupling of the eigenmodes of the empty waveguide. Due to the metal insert, a linear transformation of these matrices has to be carried out. The transformation matrices are $[\mathcal{J}^{HH}]$, $[\mathcal{J}^{EH}]$, $[\mathcal{U}^{EE}]$ and $[\mathcal{U}^L]$ with the elements $\mathcal{J}^{HH}_{pi}$, $\mathcal{J}^{EH}_{pi}$, $\mathcal{U}^{EE}_{pi}$, and $\mathcal{U}^L_{pi}$, respectively, given by (3-a)–(3-c). Note that waveguides which have the same dielectric substrate are described by the same matrices $[R^h]$, $[R^e]$, $[T]$, and $[S]$.

If all expansion coefficients in (4-a)–(4-f) except U^E_i and U^H_i are eliminated, one arrives at a proper matrix eigenvalue problem.

$$\begin{bmatrix} [I]-[\tilde{\kappa}^h]^2[\mathcal{R}^h] & -[\tilde{\kappa}^h]^2[\mathcal{T}]^t \\ [0] & [I]-[\tilde{\kappa}^e][\mathcal{S}][\tilde{\kappa}^e] \end{bmatrix} \begin{pmatrix} \boldsymbol{U}^H \\ \boldsymbol{U}^E \end{pmatrix} = \left(\tilde{\beta}\right)^2 \begin{bmatrix} [\mathcal{R}^{hh}] & [\mathcal{T}]^t \\ [\mathcal{T}] & [\mathcal{R}^{ee}] \end{bmatrix} \begin{pmatrix} \boldsymbol{U}^H \\ \boldsymbol{U}^E \end{pmatrix} \tag{8}$$

The unit matrix and the zero matrix are denoted by $[I]$ and $[0]$, respectively. The column vectors $\boldsymbol{U}^H$ and $\boldsymbol{U}^E$ have the elements U^H_n and U^E_n, respectively. The eigenvalues are the normalized sqares of the propagation constants. Since in (8) all matrix elements are real the eigenvalues are either real or complex-conjugate pairs.

NUMERICAL RESULTS

The dispersion characteristics of the dominant and higher order modes of various planar structures have been investigated by the eigenmode transformation technique.

Fig. 2 shows the cross sections of some shielded transmission lines. All these transmission lines have the same substrate. Therefore the same coupling matrices (7-a)–(7-d) could be used for all of them. The transformation matrices $[\mathcal{J}^{HH}]$, $[\mathcal{J}^{EH}]$, $[\mathcal{U}^{EE}]$ and $[\mathcal{U}^L]$, however, had to be determined for each structure, seperately. The transformation matrices of microstrip lines and coupled microstrip lines were computed by the methods, presented in [3] wheras for the computation of the transformation matrices of finlines and coplanar lines the method presented in [5] has been employed. Fig. 3 shows the dispersion characteristics of the dominant modes in the planar transmission lines shown in Fig. 2 in comparison with the results of Yamashita and Atsuki ([6]). The agreement is good. Only for high dielectric permittivities (ϵ_r=20) there are small deviations. This can be explained by the fact that the bandwidth of the coupling matrices (7-a)–(7-d) is large for a high dielectric contrast which degrade the convergence of the infinite sums involved in the calculations. Fig. 4 shows the electric field lines corresponding to a TE-eigenmode of a shielded strip line as a linear combination of the housing eigenmodes. This field representation provides the data necessary for the eigenmode transformation. The corresponding modal spectrum (two real eigenmodes forming a pair of complex modes between 16 and 25 GHz are shown in Fig. 5 agrees well with that obtained in [7].

CONCLUSIONS

A proper matrix eigenvalue formulation for the analysis of shielded waveguides containing a dielectric and metal inserts has been proposed. The metal inserts are taken into account in form of linear transforamtions for the coupling matrices corresponding to the dielectric which makes the method modular. For shielded dielectric waveguides, the numerical implementation of the matrix eigenvalue problem has been compared to other methods and the validity of the transformation technique has been checked for different types of planar transmission line.

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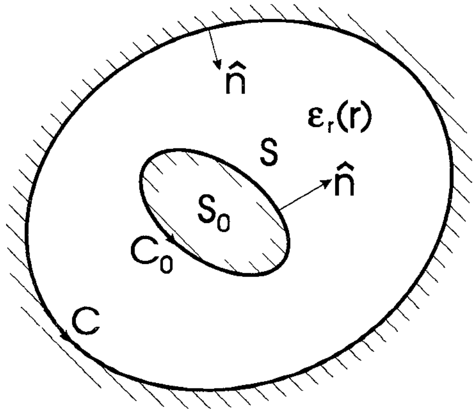

Fig. 1: Cross section of an inhomogeneously filled shielded waveguide.

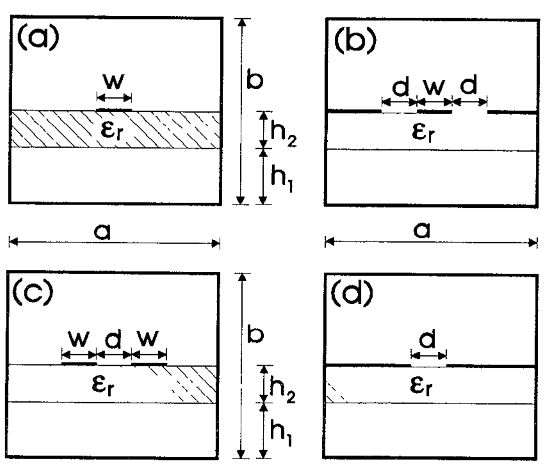

Fig. 2: Various shielded planar transmission lines with similar boundary conditions. Cross section of an inhomogeneously filled shielded waveguide.

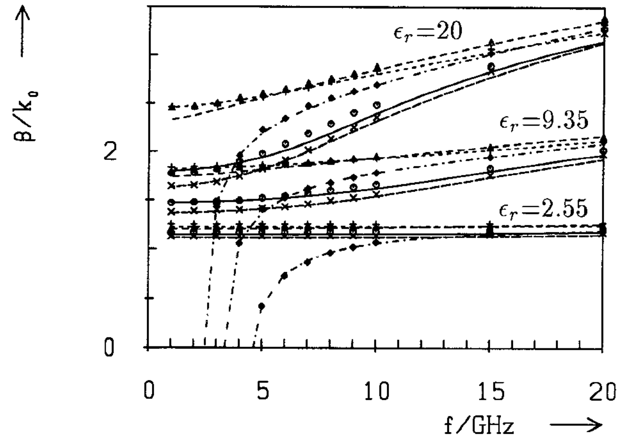

Fig. 3:
Dispersion of the dominant modes of the structures, shown in Fig. 2.
Parameters: $a = 20\,\mathrm{mm}$, $w = d = 2\,\mathrm{mm}$, $b = 10\,\mathrm{mm}$, $h_1 = 4.5\,\mathrm{mm}$, $h_2 = 1\,\mathrm{mm}$.
Presented method: —, - - -, · · ·, – –, – · –
Results of [6]: ∘, △, +, ×, ⋄
correspond to (a), (b), odd mode of (c), even mode of (c), (d), respectively, in Fig. 2.

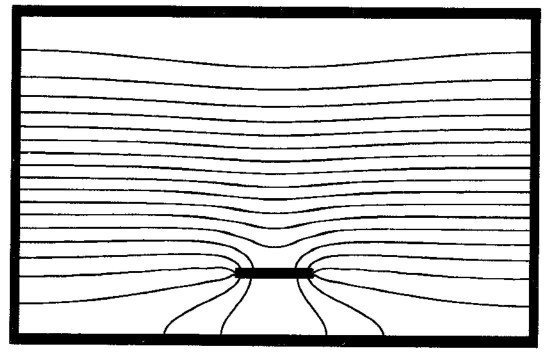

Fig. 4: Electric field of a TE mode corresponding to a shielded strip line.

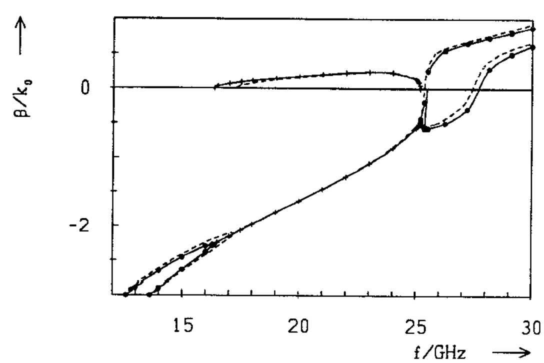

Fig. 5: Modal spectrum of a shielded microstrip line.
Parameter: $a = 10\,\mathrm{mm}$, $w = 1\,\mathrm{mm}$, $b = 5\,\mathrm{mm}$, $h_2 = 1\,\mathrm{mm}$, $h_1 = 0$, $\epsilon_r = 10$.
—: presented method, ∘(+) calculated real (complex) modes, - -: results of [7].